Mathematics of Computers

Created for Moreno Valley High School by Paul Ellsworth

Objectives:
1 Students will know and understand the process of converting numbers from the decimal numbering system to the binary numbering system.
2 Students will know and understand the process of converting numbers from the binary numbering system to the decimal numbering system.

CA Mathematics Algebra 1 Content Standard 1.0 – Students identify and use the arithmetic properties of subsets of integers and rational, irrational, and real numbers, including closure property for the four basic arithmetic operations were applicable.

CA Career Technical Education Model Curriculum Standard, Engineering Technology Pathway, D5.2 Determine what information and principals are relevant to a problem and its analysis.

CA Career Technical Education Model Curriculum Standard, Engineering Technology Pathway, D8.5 Understands the relationship among computer hardware, networks, and operating systems.

Section 4 – Decimal to Binary Conversion

Decimal to binary integer conversion is performed by subtracting the largest powers of two that is smaller than the decimal number from the decimal number. This process of successive subtraction is repeated until the decimal number is reduced to zero.

Powers of Two	
2^N	Value
2^0	1
2^1	2
2^2	4
2^3	8
2^4	16
2^5	32
2^6	64
2^7	128
2^8	256
2^9	512
2^{10}	1024
2^{11}	2048
2^{12}	4096
2^{13}	8192
2^{14}	16384
2^{15}	32768
2^{16}	65536

The direct method of decimal to binary conversion can be summarized in a step-by-step procedure.

Step 1 Successively subtract the largest possible power of two from the decimal number.
Step 2 Continue performing step 1 until a zero remainder occurs.
Step 3 Write a one for each power of two subtracted.
Step 4 Write a zero for each power of two not used.
Step 5 Write the ones and zeros in the order the subtractions are performed. All powers of

two used (from the largest through and including 2^0) must be accounted for by using either one or zero.

Example 8 – Convert 1000_{10} to its binary equivalent form.

From the above table we can see the closest power of two value that is equal to or less than 1000 is 2^9 or 512. We subtract these two numbers. 1000 – 512 = 488.

We repeat the same process for 488. We choose 2^8 or 256. 488 – 256 = 232.

Again we repeat the process for 232. We choose 2^7 or 128. 232 – 128 = 104.

Again we repeat the process for 104. We choose 2^6 or 64. 104 – 64 = 40.

Again we repeat the process for 40. We choose 2^5 or 32. 40 – 32 = 8.

Now 8 is the same as 2^3 so we use that. Notice we had to skip 2^4 as it was too large for the remaining value. Since we have a value of 0 we do not have to subtract any additional values, thus, 2^2, 2^1, and 2^0 are not used. From this information we can create the binary equivalent of the decimal number 1000.

2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
1	1	1	1	1	0	1	0	0	0

From the table we can see that $1000_{10} = 1111101000_2$

Another quicker way of doing the decimal to binary conversion is to divide the number repeatedly by two and keep track of the remainders. It is known as the successive division method. A table of the powers of two is not needed – all we need to do is divided by two.

Step 1 Successively divide the decimal number by two.
Step 2 Place the quotients directly beneath the dividend.
Step 3 Place the remainders opposite the quotients.
Step 4 The equivalent binary number becomes the remainders – the final remainder being the most significant digit (MSD) and the first remainder being the least significant digit (LSD).

2 | 25	
2 | 12	1
2 | 6	0
2 | 3	0
2 | 1	1
0	1

Taking the remainders from the bottom to the top will give us the binary equivalent of our number. In this case $25_{10} = 11001_2$. This method is quicker than the previous method.

Example 9 – Convert 188_{10} to binary.

$$\begin{array}{r|ll} 2 & 188 & 0 \\ 2 & 94 & 0 \\ 2 & 47 & 1 \\ 2 & 23 & 1 \\ 2 & 11 & 1 \\ 2 & 5 & 1 \\ 2 & 2 & 1 \\ 2 & 1 & 0 \\ & 0 & 1 \end{array}$$

Thus $188_{10} = 101111100_2$.

Section 5 – Binary to Decimal Conversion.

Binary numbers can be written in terms of their weights as was shown in Section 3. To convert the binary number to a decimal is by adding the weights together.

Example 10 – Convert 1011_2 to its decimal equivalent.

$$1011_2 = 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$$

Or

$$1011_2 = 8 + 0 + 2 + 1$$

Thus $$1011_2 = 11_{10}$$

Example 11 – Convert 101010_2 to its decimal equivalent.

$$101010_2 = 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$$
$$101010_2 = 32 + 0 + 8 + 0 + 2 + 0$$
$$101010_2 = 42_{10}$$

Exercise 2

1. Convert the following decimal numbers to their binary equivalents.

a. 5	b. 9	c. 12
d. 15	e. 17	f. 20
g. 27	h. 29	i. 33
j. 50	k. 75	l. 100
m. 127	n. 502	o. 2048

2. Convert the following binary numbers to their decimal equivalents.

a. 10	b. 110	c. 1011
d. 1101	e. 10011	f. 110010
g. 101011	h. 1110001	i. 1001001
j. 10101111	k. 11011100	l. 11101110
m. 1111100011	n. 101111110	o. 1011110011